NUMBER SYSTEMS

CONTENTS

This chapter will explore many different types of numbers and how they are classified. There will be a special focus on irrational numbers and how to use them in calculations and representation of these numbers in a number line. The first section will explain how mathematicians classify numbers-natural numbers, whole numbers, integers, rationals, irrationals and real numbers. These classifications are important to set theory and number theory. We will also explain how to arrange numbers in a number line.

Classification of Numbers

Natural Numbers

They are the positive numbers. We use these numbers to count objects. 1,2,3,4,...... are natural numbers. 1 is the smallest natural number. The letter N is used to denote natural numbers.

Whole Numbers

The whole numbers are the numbers 0,1,2,3,4 and so on. The letter 'W' is used to denote whole numbers.

Note: All natural numbers are whole numbers, but not all whole numbers are natural numbers since zero is a whole number but not a natural number.

Integers

The integers are ..... -4, -3, -2, -1, 0, 1, 2, 3, 4 ..... i.e., integers are the collection of positive numbers, negative numbers and zero. It is denoted by the symbol Z.

Note: All whole numbers are integers, but not all integers are whole numbers.

Rational Numbers

The rational numbers include all the integers, plus all fractions. Every rational number can be written in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$

$\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$ etc. are rational numbers.

The collection of rational numbers is denoted by Q.

Note 1 All natural numbers, whole numbers and integers are rationals, but not all rational numbers are natural numbers, whole numbers, or integers.

Note 2 If r and s are any two rational numbers then $\frac{r+s}{2}$ is a rational number between r and s.

> **Example** : Find a rational number between 1 and 2
>
> Rational number between 1 and 2 is $\frac{1+2}{2} = \frac{3}{2}$

Note 3 There are infinitely many rational numbers between any two given rational numbers.

Irrational Numbers

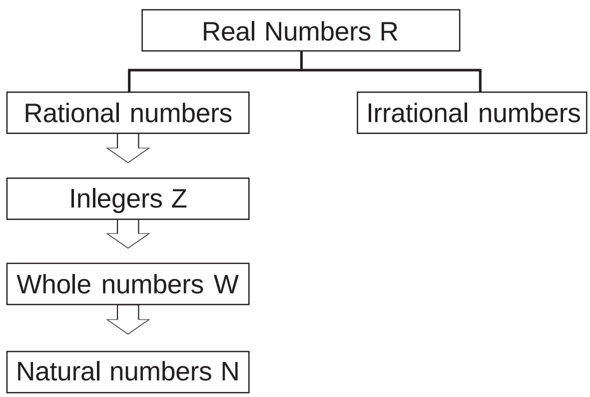
Irrational numbers are numbers which are not rationals. i.e., irrational number cannot be written in the form $\frac{p}{q}$, p and q are integers and $q \neq 0$. An irrational number is a number with a decimal that neither terminates nor repeats.

$\sqrt{2}$, $\sqrt{3}$, $\sqrt{17}$, π, 1.101101101110.... are irrational numbers.

Real Numbers

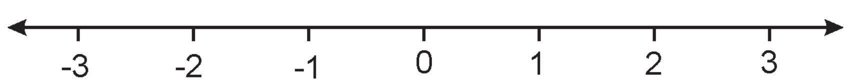
All the rational numbers and irrational numbers together form the real numbers. It is denoted by the letter R.

Our classification look loke.



Number Line

A number line is a horizontal line that has points equally spaced, which correspond to each of the real numbers.



Number line extends to right from zero and left from zero. The numbers right to the zero are positive numbers and left to the zero are negative numbers.

Note : Every real number is represented by a unique point on the number line. Also every point on the number line that represents on unique real number.

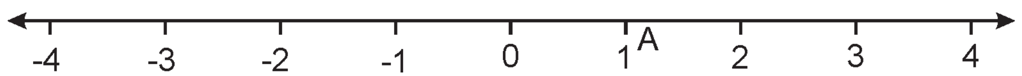
Representation of irrational numbers on the number line.

Locate $\sqrt{2}$ on the number line.

Step 1

Mark the points -2, -1, 0, 1, 2, ... on the number line.

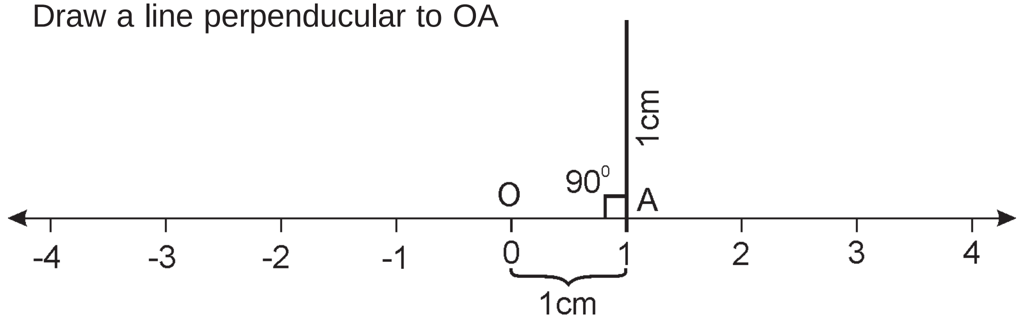
Let A denote the point 1.



We can start this procedure if the given number is less than $\sqrt{4} = 2$

Step 2

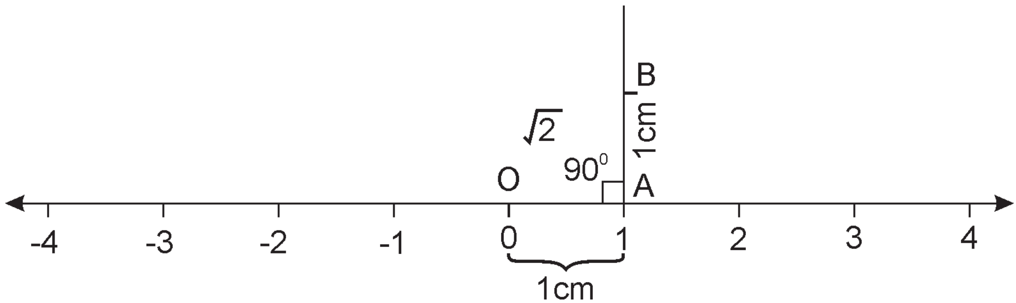
Draw a line perpenducular to OA



'O' is the point representing number zero on the number line

Step 3

Mark the point B which is 1 cm away from A.



Step 4

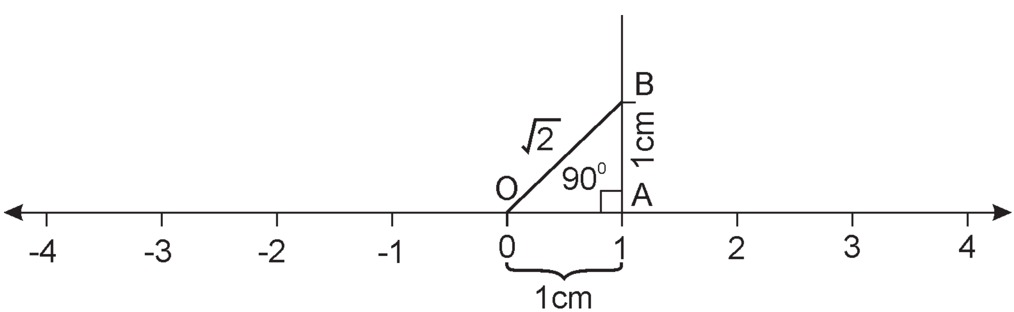
Join OB

Note: By pythagores theorem in ΔOAB

$$OB^2 = OA^2 + AB^2$$

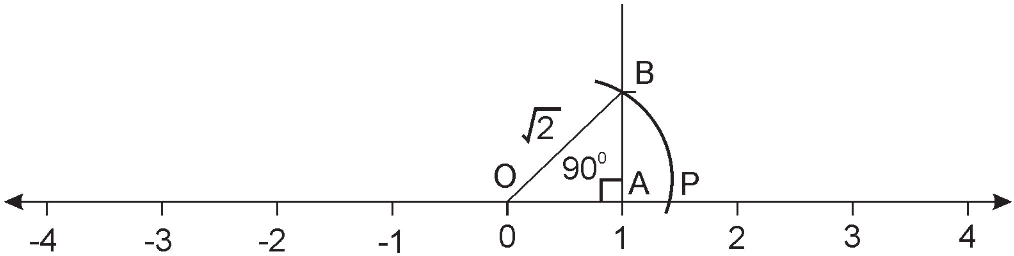
$$= 1^2 + 1^2 = 2$$

$$\therefore OB = \sqrt{2}$$



Step 5

Using a compass with centre O and radius OB, draw an arc intersecting the number line at the point P.

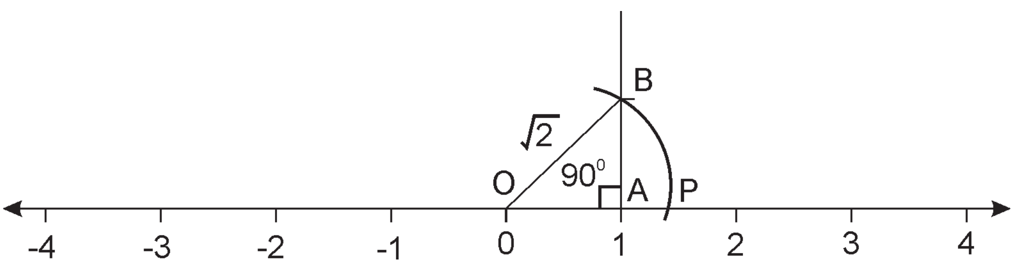


Now P corresponds to $\sqrt{2}$ on the number line

Locate $\sqrt{3}$ on the number line

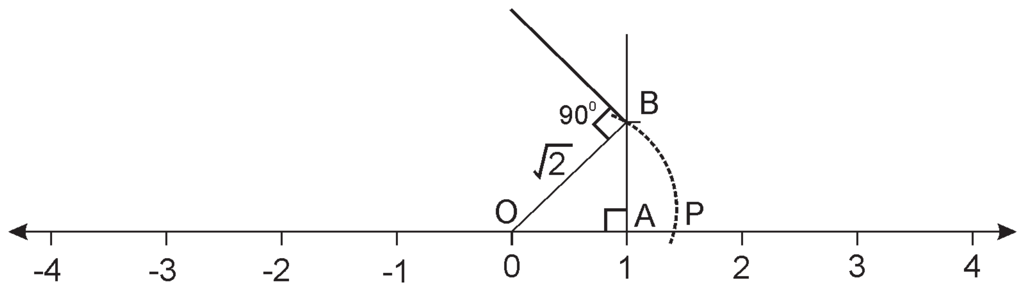
Step 1

Locate $\sqrt{2}$ on the number line as before.



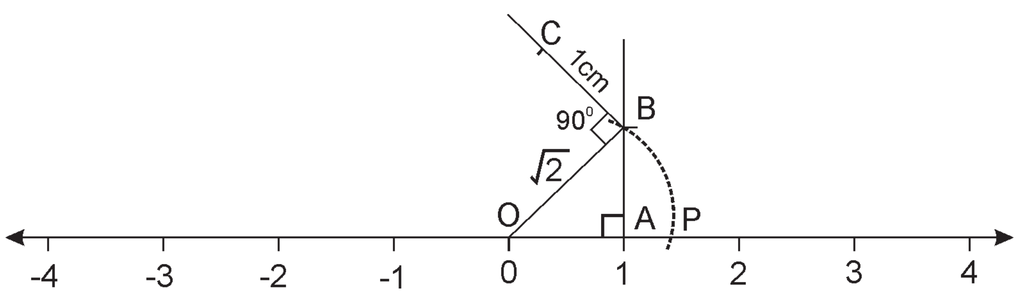
Step 2

Draw a line perpendicular to OB



Step 3

Mark the point C 1cm away from B

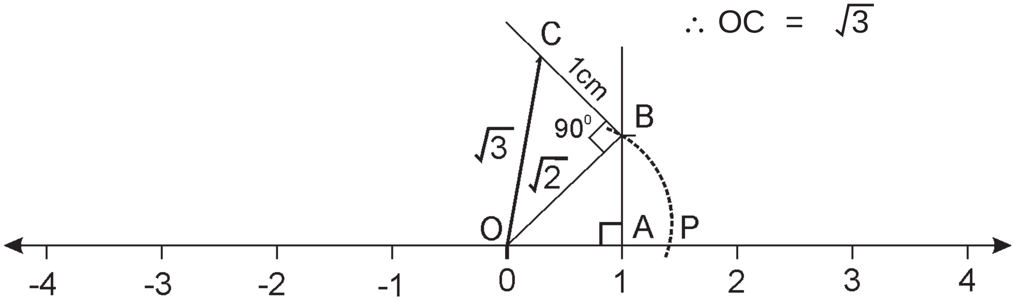


Step 4

Join OC

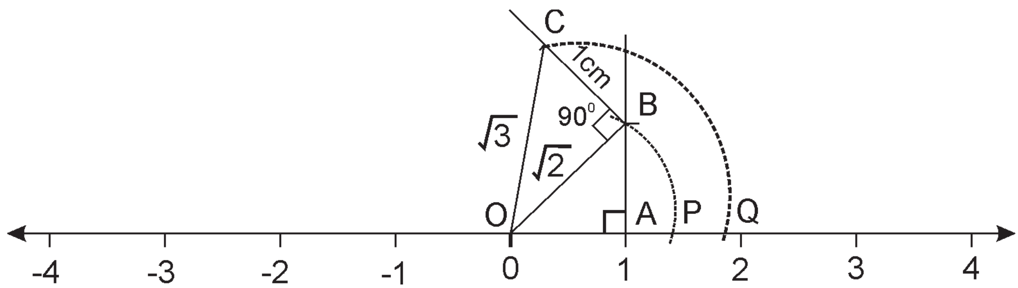
By Pythagoras theorem in ΔOBC

$$OC^2 = OB^2 + BC^2$$
$$= 2 + 1 = 3$$
$$\therefore OC = \sqrt{3}$$



Step 5

Using a compass with centre O and radius OC, draw an arc which intersect the number line at the point Q. Then Q corresponds to $\sqrt{3}$



Real numbers and their decimal expansions

You know every rational numbers (fraction) can be written as a decimal number for example, $\frac{1}{2}$ = 0.5, $\frac{1}{4}$ = 0.25 etc. In order to represent these number in decimal form, we divide the numerator by denominator. In the case of division, sometimes the remainder becomes zero and in some cases it will not become zero. Consider these cases seperately.

Case 1 : **The remainder becomes zero**

$\frac{1}{2}$ = 0.5, the remainder becomes zero after some steps.

```
    0.5
2 | 1.0
    10
    ---
     0
```

$\frac{7}{8}$ = 0.875, the remainder becomes zero after some steps.

```
     0.875
8 |  7.0
     64
     ---
      60
      56
      ---
       40
       40
       ---
        0
```

In all these cases, the decimal expansion terminates or ends after a finite number of steps. We call the decimal expansion of such numbers terminating.

Case 2 : The remainder never becomes zero

Consider the following

$\frac{1}{3} = 0.333......$

$\frac{1}{7} = 0.142857142857......$

```
     0.333
3 |  1.0
      9
      10
       9
       10
        9
         1
```

In these 2 examples, we have a repeating block of digits in the quotient. In $\frac{1}{3}$, the digit 3 is repeating in the quotient whereas in $\frac{1}{7}$, the digits 1,4,2,8,5,7 are repeating i.e., the remainder will never become zero. We call the decimal expansion of such numbers non terminating and recurring.

```
     0.142857142
7 |  1.0
      7
      30
      28
       20
       14
        60
        56
         40
         35
          50
          49
           10
            7
            30
            28
             20
             14
              6
```

Notation for non terminating, recurring decimals.

We have seen that 3 is repeating endlesly in the decimal representation of $\frac{1}{3}$. Its not possible to write all the 3's in the decimal expansion. So to represent these decimals, put a bar on repeating digits.

i.e., $\frac{1}{3} = 0.\overline{3}$ which means 3 is repeating

$\frac{1}{7} = 0.\overline{142857}$

Also 3.57272 = $3.5\overline{72}$ here 72 is repeating.

Rational numbers and their decimal expansions

The decimal expansion of a rational number is either terminating or non-terminating recurring.

Example : $\frac{1}{3} = 0.\overline{3}$ is rational

Irrational numbers and their decimal expansion

The decimal expansion of an irrational number is non terminating non-recurring. Or a number whose decimal expansion is non-terminating non-recurring is irrational.

Example : $\sqrt{2}$ = 1.4142135623.... is irrational

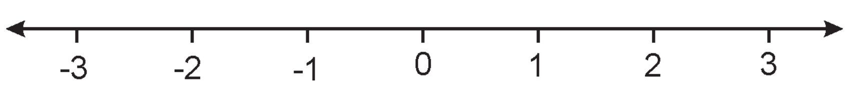
Representing Real Numbers on the number line

We have seen that every real numbers has a decimal expansion. This helps us to represent it on a number line.

Illustration : Locate 3.664 on the number line.

Step 1

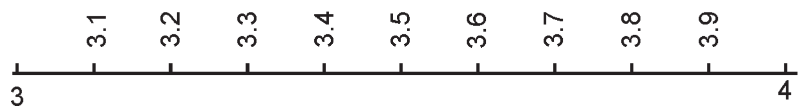
Make the points -2,-1,0,1,2,3,4 etc on the number line.



We looking closely at the number 3.664 one can see that this lies between 3 and 4

Step 2

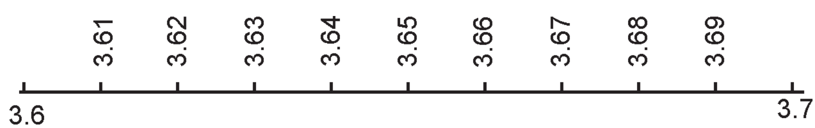
We divide the section 3 to 4 into 10 equal parts and make the points 3.1,3.2,..... 3.9 and 4



Now the points 3.664 lies between 3.6 and 3.7 there equal.

Step 3

Divide the section 3.6 to 3.7 into 10 equal parts and make the points 3.61, 3.62, ... 3.69 and 3-7



The point 3.664 lies between 3.66 and 3.67

Step 4

Divide the section 3.66 to 3.67 into 10 equal parts and make the points 3.661, 3.662, ... 3.669 and 3-67

Now we located the points 3.664 on the number line

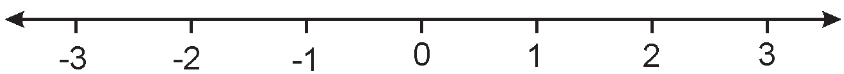
Illustration 2 : Visualize the representation of $4.3\overline{7}$ on the number line upto 5 decimal places

$4.3\overline{7}$ = 4.3777

= 4.37777 upto decimal places

Step 1

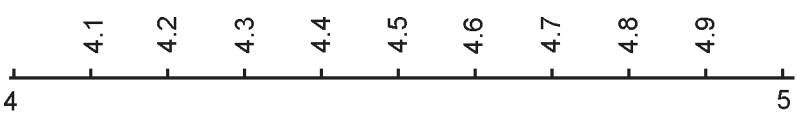
Make the points -2,-1,0,1,2,3,4 on the number line



4.37777 lies between 4 and 5

Step 2

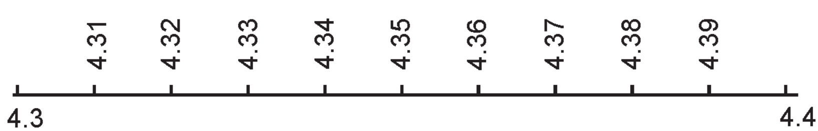
We divide the section 4 to 5 into 10 equal parts and make the points 4.1, 4.2, ... 4.9, 5.



Now 4.37777 lies between 4.3 and 4.4

Step 3

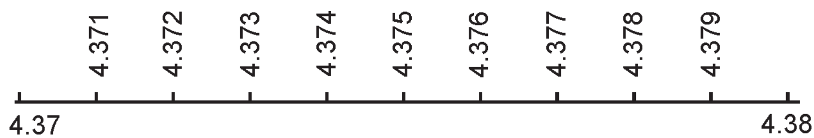
We divide the section 4.3 to 4.4 into 10 equal parts and make the points 4.31, 4.32, ... 4.39, 4.4



4.37777 lies between 4.37 and 4.38

Step 4

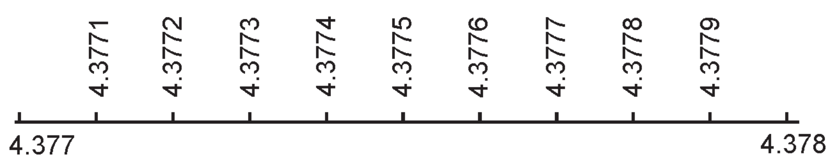
Divide the section 4.37 - 4.38 into 10 equal parts abd make the points 4.371, 4.379, 4.38



4.3777 lies between 4.377 and 4.378

Step 5

Divide the section 4.377 - 4.378 into 10 equal parts abd make the points 4.3771, 4.3772,........... 4.3779, 4.378



Now we can locate 4.37777

Actually $4.3\overline{7}$ located near to 4.37778 than 4.37777

Operations on Real Numbers

Addition

We cannot add any 2 irrational numbers. We can add only if the irrational part is same.

Example : $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$

Substraction

As in the case of addition if we want to substract 2 real numbers the irrational parts must be same.

Example : $5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$

Multiplication

We can find the product of any 2 real numbers.

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

Example : $\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$

Division

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

Example : $\frac{\sqrt{6}}{\sqrt{3}} = \sqrt{\frac{6}{3}} = \sqrt{2}$

Properties

- The sum of a rational number and an irrational number is irrational.

Example : 3 is a rational number, $\sqrt{2}$ is an irrational
the sum $3 + \sqrt{2} = 3 + \sqrt{2}$ which is an irrational number

- The difference of a rational number and an irrational number is irrational

Example : 5 is a rational, $\sqrt{3}$ is irrational
the difference $5 - \sqrt{3}$ is irrational

- The product of a non-zero rational number with an irrational number is irrational

Example : 5 is a rational, $\sqrt{3}$ is irrational
the product 5 x $\sqrt{3}$ is irrational

- The quotient of non-zero rational number with an irrational number is irrational

Example : 5 is a rational, $\sqrt{2}$ is irrational
the quotient $\frac{5}{\sqrt{2}}$ is irrational

- If we add any two irrationals, the result may be rational or irrational

Example : $5\sqrt{2}$, $4\sqrt{2}$ are irrational numbers
the sum $5\sqrt{2} + 4\sqrt{2} = 9\sqrt{2}$ is an irrational number
$\sqrt{2}$, $-\sqrt{2}$ are irrational numbers
the sum $\sqrt{2} + -\sqrt{2} = 0$ which is a rational number

- If we substract any 2 irrationals, the result may be rational or irrational

Example : $5\sqrt{2}$, and $4\sqrt{2}$ are irrational numbers
the difference $5\sqrt{2} - 4\sqrt{2} = \sqrt{2}$, an irrational numbers
$\sqrt{2}$, $\sqrt{2}$ are irrational numbers
the difference $\sqrt{2} - \sqrt{2} = 0$, a rational number

- If we multiply any 2 irrationals, the result may be rational or irrational.

Example : $\sqrt{2}$, $\sqrt{3}$ are irrational numbers
the product $\sqrt{2} \times \sqrt{3} = \sqrt{6}$, an irrational number
$4\sqrt{2}$, $2\sqrt{2}$ are irrational numbers
the product $4\sqrt{2} \times 2\sqrt{2} = 8(\sqrt{2})^2 = 8 \times 2 = 16$, a rational number

- If we divide any 2 irrationals, the result may be rational or irrational.

Example : $\sqrt{6}$, $\sqrt{2}$ are irrational numbers
the quotient $\frac{\sqrt{6}}{\sqrt{2}} = \sqrt{3}$, an irrational numbers
$2\sqrt{2}$, $\sqrt{2}$ are irrational numbers, the quotient
$\frac{2\sqrt{2}}{\sqrt{2}} = 2$, a rational number.

Identities

1. $(a + b)(a - b) = a^2 - b^2$

Example : $(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2}) = (\sqrt{3})^2 - (\sqrt{2})^2 = 3 - 2 = 1$

2. $(a + b)(c + d) = ac + ad + bc + bd$

Example : $(\sqrt{2}+\sqrt{3})(\sqrt{7}+\sqrt{5}) = \sqrt{2 \times 7} + \sqrt{2 \times 5} + \sqrt{3 \times 7} + \sqrt{3 \times 5}$
$= \sqrt{14} + \sqrt{10} + \sqrt{21} + \sqrt{15}$

4. $(a + b)^2 = a + 2ab + b$

Example : $(\sqrt{2} + \sqrt{3})^2 = (\sqrt{2})^2 + 2\sqrt{2} \times \sqrt{3} + (\sqrt{3})^2$
$= 2 + 2\sqrt{6} + 3 = 5 + 2\sqrt{6}$

Conjugate pairs (Surds)

$\sqrt{a} + \sqrt{b}$ and $\sqrt{a} - \sqrt{b}$ are conjugate pairs. If we multiply 2 conjugate pairs, the product will be rational number.

Example : $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are conjugate pairs

If the product of two irrational numbers is rational, each is called the **rationalising factor** of the other.

Example : Consider $\sqrt{3}$

If we multiply $\sqrt{3}$ by $\sqrt{3}$, the product is $\sqrt{3} \times \sqrt{3} = 3$

which is a rational number: $\sqrt{3}$ is the rationalising factor of $\sqrt{3}$

Rationalisation

The process of making either the denominator or numerater to a rational number is called rationalisation.

Illustration 1

Rationalise the denominater of $\frac{1}{\sqrt{3}}$

Step 1

Divide and multiply given number by $\sqrt{3}$

$\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\sqrt{3}$ is the rationalising factor of $\sqrt{3}$

Step 2

Simplyfy the expression in step1, if necessary

$\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\sqrt{3} \times \sqrt{3} = (\sqrt{3})^2 = 3$

Now the fraction becomes $\frac{\sqrt{3}}{3}$. i.e., the denominator is a rational number.

Illustration 2

Rationalise the denominater of $\frac{\sqrt{5}}{3 - \sqrt{2}}$

$\frac{\sqrt{5}}{3 - \sqrt{2}}$	Given
$\frac{\sqrt{5}}{3 - \sqrt{2}} \times \frac{3 + \sqrt{2}}{3 - \sqrt{2}}$	Multiply the numerator and denominater by $3 + \sqrt{2}$
$\frac{\sqrt{5}(3 + \sqrt{2})}{3^2 - (\sqrt{2})^2}$	$(a + b)(a - b) = a^2 - b^2$ $(3 + \sqrt{2})(3 - \sqrt{2}) = 3^2 - (\sqrt{2})^2$

If the denominater is of the form $\sqrt{a} + \sqrt{b}$ or $\sqrt{a} - \sqrt{b}$ multiply and divide by its conjugate pairs

$\frac{\sqrt{5}(3+\sqrt{2})}{9-2}$	$3^2 = 9$ and $(\sqrt{2})^2 = 2$
$\frac{\sqrt{5}(3+\sqrt{2})}{7}$	Simplify the denominator
$\frac{\sqrt{5} \times 3 + \sqrt{5} \times \sqrt{2}}{7}$	Open the brackets
$\frac{3\sqrt{5} + \sqrt{10}}{7}$	$\sqrt{a} \times \sqrt{b} = ab$ $\sqrt{5} \times \sqrt{2} = \sqrt{10}$

Now the denominator is a rational number.

Laws of exponents for real numbers

The following laws will be helpful to simplify the expressions.

Let a>0 be a real number and m and n are rational numbers.

1. $a^m \; a^n = a^{m+n}$
2. $(a^m)^n = a^{mn}$
3. $\frac{a^m}{a^n} = a^{m-n}$, m>n, $a \neq 0$
4. $a^m \; b^m = (ab)^m$

Descriptive Problems

1. Represent as a decimal number

   a) $\frac{4}{15}$ b) $\frac{2}{7}$ c) $\frac{4}{9}$

Answer

a) $\frac{4}{15} = 0.266....$

$= 0.2\overline{6}$

```
      0.266
 15 | 4.0
      30
      ---
       100
        90
       ---
        100
         90
        ---
         10
```

b) $\frac{2}{7} = 0.\overline{285714}$

```
      0.28571428
  7 | 20
      14
      --
       60
       56
       --
        40
        35
        --
         50
         49
         --
          10
           7
          --
           30
           28
           --
            20
            14
            --
             60
             56
             --
              4
```

c) $\frac{4}{9} = 0.\overline{4}$

```
      0.44
  9 | 40
      36
      --
       40
       36
       --
        4
```

2. Insert two rational numbers between

a) 3 and 4 b) $\frac{1}{3}$ and $\frac{1}{4}$

Answer

a) The rational numbers between a and b is $\frac{a+b}{2}$

$\therefore$ rational number between 3 and 4 = $\frac{3+4}{2} = \frac{7}{2} \Rightarrow$ First rational number

$\therefore\ 3 < \frac{7}{2} < 4$

2nd rational numbers

rational number between 3 and $\frac{7}{2}$ $= \frac{3+\frac{7}{2}}{2} = \frac{6+7}{2 \times 2}$

$= \frac{13}{4}$

$3 < \frac{13}{4} < \frac{7}{2} < 4$

i.e., $\frac{13}{4}$ and $\frac{7}{2}$ are 2 rational numbers between 3 and 4

b) Rational number between $\frac{1}{3}$ and $\frac{1}{4}$ $= \frac{\frac{1}{3}+\frac{1}{4}}{2}$

$= \frac{4+3}{12 \times 2}$

$= \frac{7}{24}$

Inserting one rational number between $\frac{1}{3}$ and $\frac{1}{4}$

i.e., $\frac{1}{3} < \frac{7}{24} < \frac{1}{4}$

Rational number between $\frac{1}{3}$ and $\frac{7}{24}$ $= \dfrac{\frac{1}{3}+\frac{7}{24}}{2}$

$= \frac{8 + 7}{24 \times 2}$

$= \frac{15}{48}$

i.e., $\frac{1}{3} < \frac{15}{48} < \frac{7}{24} < \frac{1}{4}$

$\frac{15}{48}$ and $\frac{7}{24}$ are 2 rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$

Note : The above insertion of rational number is not unique, as there are infinitive rational numbers between any two given rational numbers.

3. Express the following in the form $\frac{p}{q}$

a) $0.\overline{4}$ b) $0.\overline{45}$ c) $5\overline{65}$

Answer

a) Let x = 0.4 = 0.444.... -------- (1)

Since one digit is repeating, multiply x by 10.

$10x$ = (0.44...) x 10

$10x$ = 4.4.... -------- (2)

(2) - (1) ⇒ $10x - x$ = 4.44... – 0.44...

$9x$ = 4 + 0.44.... – 0.44

4.44.... = 4 + 0.44...

$$9x = 4$$

$$\Rightarrow x = \frac{4}{9}$$

b) Let x = 0.4545... -------- (1)

Since 2 digit are repeating, multiply x by 100

$$100x = (0.4545...) \times 100$$

$$100x = 45.4545... \text{ ------- } (2)$$

(2) - (1) $100x - x = 45.4545... - 0.4545$

$$= 45 + 0.4545... - 0.4545$$

45.4545... = 45+04545...

$$99x = 45$$

$$x = \frac{45}{99}$$

c) Let x = 0.565565 ...... --------- (1)

Since 3 digits are repeating, multiply x by 1000

$$1000x = (0.565...) \times 1000$$

$$= 565.565565.... \text{ --------- } (2)$$

(2) - (1) $\Rightarrow$

$$1000x - x = 565.565565.... - 0.565...$$

$$= 565 + 0.565565 - 0.565...$$

565.565...= 565 + 0.565...

$$999x = 565$$

$$x = \frac{565}{999}$$

4. Rationalise the denominator of $\frac{3+\sqrt{7}}{3-4\sqrt{7}}$

Answer

Consider the given number $\frac{3+\sqrt{7}}{3-4\sqrt{7}}$

Since the denominator is of the form $a-\sqrt{b}$

Multiply and divide the given number by the conjugate pair of $3-4\sqrt{7}$

So multiply and divide by $3+4\sqrt{7}$

$$\Rightarrow \frac{3+\sqrt{7}}{3-4\sqrt{7}} \times \frac{3+4\sqrt{7}}{3+4\sqrt{7}}$$

$$\Rightarrow \frac{(3+\sqrt{7})(3+4\sqrt{7})}{3^2-(4\sqrt{7})^2} = \frac{(3+\sqrt{7})(3+4\sqrt{7})}{9-16 \times 7} = \frac{(3+\sqrt{7})(3+4\sqrt{7})}{9-112}$$

$$= \frac{(3+\sqrt{7})(3+4\sqrt{7})}{-103}$$

$$= \frac{3(3)+3 \times 4\sqrt{7}+3\sqrt{7}+4\sqrt{7}.\sqrt{7}}{-103}$$

$$= \frac{9+12\sqrt{7}+3\sqrt{7}+28}{-103}$$

$$= \frac{37+15\sqrt{7}}{-103}$$

$$= \frac{-37 - 15\sqrt{7}}{103}$$

5. Simplify $\left(\frac{9}{25}\right)^{\frac{-3}{2}}$

Answer

Consider $\left(\frac{9}{25}\right)^{\frac{-3}{2}} = \left(\frac{25}{9}\right)^{\frac{3}{2}}$ $\left\{\text{Since } \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}\right\}$

$$= \left[\left(\frac{25}{9}\right)^{\frac{1}{2}}\right]^3$$

$$= \left[\left(\frac{5^2}{3^2}\right)^{\frac{1}{2}}\right]^3$$

$$= \left[\left(\left(\frac{5}{3}\right)^2\right)^{\frac{1}{2}}\right]^3$$

$$= \left[\left(\frac{5}{3}\right)^{2 \times \frac{1}{2}}\right]^3$$

$$= \left(\frac{5}{3}\right)^3 = \frac{5^3}{3^3} = \frac{125}{27}$$

Training for Competitive Examinations

Very challenging problems are marked with ✹

1 Which of the following numbers lies between $\frac{1}{9}$ and $\frac{7}{9}$?

a) $\frac{6}{7}$ b) $\frac{4}{5}$ c) $\frac{4}{9}$ d) $\frac{1}{19}$

2. Which of the following numbers is a natural number?

a) -2 b) -1 c) 0 d) 1

3. What is the decimal equivalent of $\frac{1}{7}$?

a) $0.\overline{142857}$ b) $0.\overline{141857}$ c) $0.\overline{14857}$ d) $0.\overline{74857}$

4. $\frac{-27}{-3}$ is a

a) Negative rational number

b) Positive rational number

c) either positive or negative rational number

d) neither positive and negative rational number

5. $\frac{\sqrt{5}-4}{\sqrt{5}+4}$ is

a) rational number b) an irrational number

c) an integel d) a natural number

6. Value of $14.2\overline{7}$ is

a) $\frac{247}{28}$ b) $\frac{257}{18}$ c) $\frac{237}{38}$ d) $\frac{227}{48}$

7. If $\frac{3-\sqrt{5}}{3+2\sqrt{5}} = a\sqrt{5} - b$ when a and b are rational numbers then

a) $a = \frac{9}{11}$ $b = \frac{19}{11}$ b) $a = \frac{19}{11}$ $b = \frac{9}{11}$

c) $a = \frac{10}{11}$ $b = \frac{21}{11}$ d) $a = \frac{2}{11}$ $b = \frac{8}{11}$

8. The product of $\sqrt[3]{4}$ and $\sqrt[3]{22}$ is

a) $2\sqrt[3]{11}$ b) $\sqrt[3]{11}$ c) $4\sqrt[3]{11}$ d) none of these

9. The value of $\left(\sqrt[6]{27} - \sqrt{6\frac{3}{4}}\right)^2$ is

a) $\frac{\sqrt{3}}{2}$ b) $\frac{3}{2}$ c) $\frac{\sqrt{3}}{4}$ d) $\frac{3}{4}$

10. The rationalising factor of $\sqrt[9]{2048}$ is

a) $2^{\frac{7}{9}}$ b) $2^{\frac{4}{9}}$ c) $2^{\frac{2}{9}}$ d) $2^{\frac{8}{9}}$

11. Which of the following pairs of numbers has 2 numbers, the difference of which will be an irrational number?

a) $3\sqrt{7}, 3\sqrt{7}$ b) 2, -2 c) $3\sqrt{2}, -3\sqrt{2}$ d) $\sqrt{8}, \sqrt{8}$

12. π is

a) rational number b) irrational number

c) imaginary d) integer

13. The number $(\sqrt{3}+\sqrt{5})^2$ is

a) rational number b) irrational number

c) can't say d) none of these

14 The fraction $\frac{2(\sqrt{2}+\sqrt{6})}{3(\sqrt{2+\sqrt{3}})}$ has a value equal to

a) $2\sqrt{2}$ b) 1 c) $\frac{2\sqrt{2}}{3}$ d) $\frac{4}{3}$

15. Which of the following pairs of numbers has 2 numbers, the product of which will be a rational number?

a) $2-\sqrt{3}, 2+\sqrt{3}$ b) $3-\sqrt{3}, 4-\sqrt{3}$ c) $2-\sqrt{7}, 2\sqrt{7}$ d) $1-\sqrt{3}, 7$

16. Which of the following statement is false?

a) all natural numbers are integers

b) all whole numbers are natural numbers

c) all whole numbers are integers

d) all irrational numbers are real numbers.

17. Which of the following statement is false

a) Smallest natural numbers is 1

b) Largest integer cannot be determined

c) Smallest integer is 0

d) There are infinite numbers lying between 2 numbers.

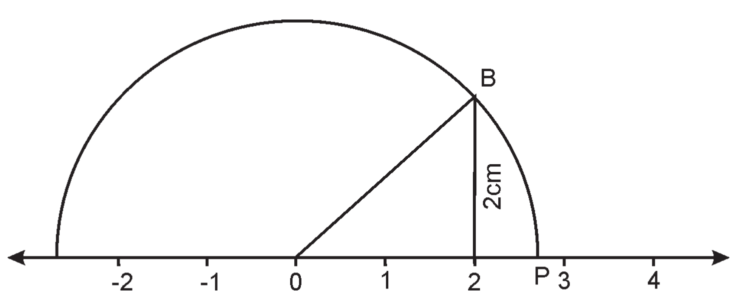
18. Which of the following irrational numbers is represented by the following diagram?

a) $2\sqrt{2}$

b) $\sqrt{3}$

c) $\sqrt{7}$

d) $\sqrt{11}$



19. Which of the following statement is true?

a) Every irrational number is a real number

b) Square roots of all integers are real

c) every real number is an irrational number

d) every point on the number line is of the form $\sqrt{n}$.

where n is a natural number

20. If $\frac{-4}{x} = \frac{x}{4}$, then x is

a) rational number b) irrational number

c) natural numbers d) none of these

21 The value of $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{8}} + \frac{1}{\sqrt{8}+\sqrt{9}}$ is

a) 0 b) 1 c) 2 d) 4

22 What is the percentage of least number in the greatest number if $\frac{3}{6}$, $\frac{9}{6}$, $\frac{1}{6}$, $\frac{7}{6}$ are arranged in ascending order?

a) $11\frac{1}{9}$ % b) 20% c) 10% d) 25%

23. If $x = 3 + \sqrt{8}$, then $x^3 + \frac{1}{x^3} =$

a) 216 b) 198 c) 192 d) 261

24. The average of the middle two rational numbers if $\frac{4}{7}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{5}{9}$ are aranged in asending order is

a) $\frac{86}{90}$ b) $\frac{86}{45}$ c) $\frac{45}{45}$ d) $\frac{43}{90}$

25. Which of the following is not irrational?

a) $\pi - (-\pi)$ b) $\sqrt{5} + \sqrt{4}$

c) $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})$ d) $(5 + \sqrt{3})(\sqrt{5})$

26. The product of $\sqrt[3]{4}$ and $\sqrt[3]{44}$ is

a) $\sqrt[3]{176}$ b) $\sqrt[3]{11}$ c) $\sqrt[4]{11}$ d) none of these

27 If $x = \frac{\sqrt{5} - 2}{\sqrt{5} + 2}$ and $y = \frac{\sqrt{5} + 2}{\sqrt{5} - 2}$ then $x^2 + y^2 + xy =$

a) 161 b) $161 - 72\sqrt{3}$ c) $161 + 72\sqrt{5}$ d) 323

28 Find the values of a and b if $\frac{5 + 3\sqrt{2}}{5 - 3\sqrt{2}} = a + b\sqrt{2}$

29. Find the least rationalising factor of $\sqrt{18} - \sqrt{50}$

a) $\sqrt{18} + \sqrt{50}$ b) $\sqrt{2}$ c) $\sqrt{18}$ d) $\sqrt{50}$

30 Simplify : $\frac{22}{2\sqrt{3} + 1} + \frac{17}{2\sqrt{3} - 1} =$

a) $\frac{78 - 5\sqrt{3}}{11}$ b) $\frac{78 + 5\sqrt{3}}{11}$

c) $\frac{78\sqrt{3} + 5}{11}$ d) $\frac{78\sqrt{3} - 5}{11}$

Answers

1. c

It is clear that $\frac{1}{9} < \frac{4}{9} < \frac{7}{9}$

2. d

3. a

$$\frac{1}{7} = 0.142857142857...$$

$$= 0.\overline{142857}$$

$$
\begin{array}{r}
0.1428571 \\
7 \,\overline{)\,10} \\
7 \\
\hline
30 \\
28 \\
\hline
20 \\
14 \\
\hline
60 \\
56 \\
\hline
40 \\
35 \\
\hline
50 \\
49 \\
\hline
10
\end{array}
$$

4. b

$$\frac{-27}{-3} = 9 = \frac{9}{1}$$

So it is a positive rational number

5. b

$$\frac{\sqrt{5} - 4}{\sqrt{5} + 4} = \frac{\sqrt{5} - 4}{\sqrt{5} + 4} \times \frac{\sqrt{5} - 4}{\sqrt{5} - 4}$$

$$= \frac{(\sqrt{5} - 4)^2}{(\sqrt{5})^2 - 4^2} = \frac{(\sqrt{5} - 4)^2}{5 - 16}$$

$$= \frac{(\sqrt{5} - 4)^2}{-11}$$

$$= \frac{(\sqrt{5})^2 - 2\sqrt{5}(4) + 4^2}{-11}$$

$(a - b)^2 = a^2 - 2ab + b^2$

$$= \frac{5 - 8\sqrt{5} + 16}{-11}$$

Since there is an irrational number $\sqrt{5}$ in the 2nd term after simplification, its an irrational number

6. b

Let $x = 14.2\overline{7} = 14.277...$ --------- (1)

Since the one digit is repeating, multiply x by 10

$10x = (14.277...)\ (10)$

$10x = 142.\ 77...$ --------- (2)

(2) - (1) $\Rightarrow$

$10x - x = 142.77... - 14.277$

$9x = 128.5$

$$\begin{array}{r} 142.777... \\ 14.277... \\ \hline 128.500 \end{array}$$

$$x = \frac{128.5}{9}$$

$$= \frac{1285}{10 \times 9}$$

$$= \frac{1285}{90} = \frac{257}{18}$$

7. a

$$\frac{3-\sqrt{5}}{3+2\sqrt{5}} = \frac{3-\sqrt{5}}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$$

$$= \frac{(3-\sqrt{5})(3-2\sqrt{5})}{3^2-(2\sqrt{5})^2}$$

$$= \frac{(3-\sqrt{5})(3-2\sqrt{5})}{9-4 \times 5}$$

$$= \frac{(3-\sqrt{5})(3-2\sqrt{5})}{9-20}$$

$$= \frac{(3-\sqrt{5})(3-2\sqrt{5})}{-11}$$

$$= \frac{3(3)-3(2\sqrt{5})-3\sqrt{5}+2\sqrt{5}(\sqrt{5})}{-11}$$

$$= \frac{9-6\sqrt{5}-3\sqrt{5}+10}{-11} = \frac{19-9\sqrt{5}}{-11}$$

$$= \frac{-19}{11} + \frac{9\sqrt{5}}{11}$$

Given $\frac{3-\sqrt{5}}{3+2\sqrt{5}} = a\sqrt{5} - b$

i.e., $\frac{-19}{11} + \frac{9\sqrt{5}}{11} = a\sqrt{5} - b$

Equating the coefficient of $\sqrt{5}$

$$\frac{9}{11} = a$$

Equating constant term,

$$\frac{-19}{11} = -b \text{ or } b = \frac{19}{11}$$

$$\text{i.e., } a = \frac{9}{11} \text{ and } b = \frac{19}{11}$$

8. a

$$\sqrt[3]{4} = 4^{\frac{1}{3}}$$

$$\sqrt[3]{22} = 22^{\frac{1}{3}}$$

$$\therefore \text{ product} = \sqrt[3]{4} . \sqrt[3]{22}$$

$$= 4^{\frac{1}{3}} \quad 22^{\frac{1}{3}}$$

$$= (4 \times 22)^{\frac{1}{3}}$$

$$= (88)^{\frac{1}{3}}$$

$$= \sqrt[3]{88}$$

$$= 2. \sqrt[3]{11}$$

$88 = 2 \times 2 \times 2 \times 11$

$\sqrt[3]{88} = 2 \sqrt[3]{11}$

2	88
2	44
2	22
	11

9. d

$$\sqrt[6]{27} = 27^{\frac{1}{6}} = (3^3)^{\frac{1}{6}}$$

$$= 3^{\frac{3 \times 1}{6}}$$

$$= 3^{\frac{1}{2}} = \sqrt{3}$$

$$\sqrt{6\frac{3}{4}} = \sqrt{\frac{27}{4}} = \sqrt{\frac{3 \times 3 \times 3}{2 \times 2}}$$

$$= \frac{3}{2}\sqrt{3}$$

$$\left(\sqrt[6]{27} - \sqrt{6\frac{3}{4}}\right) = \sqrt{3} - \frac{3}{2}\sqrt{3}$$

$$= \sqrt{3}\left(1 - \frac{3}{2}\right)$$

$$= \sqrt{3}\,\frac{(2-3)}{2}$$

$$= \sqrt{3}\,\frac{(-1)}{2} = -\frac{\sqrt{3}}{2}$$

$$\left(\sqrt[6]{27} - \sqrt{6\frac{3}{4}}\right)^2 = \left(\frac{-\sqrt{3}}{2}\right)^2 = \frac{3}{4}$$

10. a

$$\sqrt[9]{2048} = (2048)^{\frac{1}{9}}$$

$$= (2^{11})^{\frac{1}{9}}$$

$$= 2^{\frac{11}{9}}$$

$2048 = 2^{11}$

2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2

Now we have to find out the rationalising factor. i.e., if we multiply that number with $2^{\frac{11}{9}}$ the result will be a rational number.

Consider the first option, $2^{\frac{7}{9}}$

$$2^{\frac{7}{9}} . 2^{\frac{11}{9}} = 2^{\frac{18}{9}} = 2^2 = 4$$

In this case the result is a rational number

$\Rightarrow$ $2^{\frac{7}{9}}$ is a rationalising factor of $2^{\frac{11}{9}}$

11. c

Lets find out the difference of given numbers

Consider (a)

$$3\sqrt{7} - 3\sqrt{7} = 0, \text{ a rational number}$$

Consider (b)

$$2 - (-2) = 2 + 2 = 4, \text{ a rational number}$$

Consider (c)

$$3\sqrt{2} - (-3\sqrt{2}) = 3\sqrt{2} + 3\sqrt{2} = 6\sqrt{2}, \text{ an irrational number}$$

So option (c) is the answer.

12. b

π is considered as an irrational number

13. b

$$(\sqrt{3} + \sqrt{5})^2 = (\sqrt{3})^2 + 2(\sqrt{3})(\sqrt{5}) + (\sqrt{5})^2$$

$$= 3 + 2\sqrt{15} + 5$$

$$= 8 + 2\sqrt{15}$$

Since the irrational number $\sqrt{15}$ is present in the expanded form, the given expression is an irrational number.

14. d

$$\frac{2(\sqrt{2} + \sqrt{6})}{3(\sqrt{2} + \sqrt{3})} = \frac{2(\sqrt{2} + \sqrt{2}.\sqrt{3})}{3(\sqrt{2} + \sqrt{3})}$$

$$= \frac{2.\sqrt{2}\,(1+\sqrt{3})}{3(\sqrt{2+\sqrt{3}})}$$ (Taking $\sqrt{2}$ outside)

$$= \frac{2\sqrt{2}}{3}\ \frac{(1+\sqrt{3})}{\sqrt{2+\sqrt{3}}}\ \frac{(\sqrt{2-\sqrt{3}})}{\sqrt{2-\sqrt{3}}}$$ (multiply and divide by $\sqrt{2-\sqrt{3}}$)

$$= \frac{2\sqrt{2}}{3}\ \frac{(1+\sqrt{3})\,(\sqrt{2-\sqrt{3}})}{\sqrt{(2+\sqrt{3})\text{x}(2-\sqrt{3})}}$$ (Since $\sqrt{ab} = \sqrt{a}.\sqrt{b}$)

$$= \frac{2\sqrt{2}}{3}\ \frac{(1+\sqrt{3})\,(\sqrt{2-\sqrt{3}})}{\sqrt{2^2-(\sqrt{3})^2}}$$ ($(a+b)(a-b) = a^2 - b^2$)

$$= \frac{2\sqrt{2}}{3}\ \frac{(1+\sqrt{3})\,(\sqrt{2-\sqrt{3}})}{\sqrt{4-3}}$$

$$= \frac{2\sqrt{2}}{3}\ \frac{(1+\sqrt{3})\,(\sqrt{2-\sqrt{3}})}{\sqrt{1}}$$

$$= \frac{2\sqrt{2}}{3}\ (1+\sqrt{3})\,(\sqrt{2-\sqrt{3}})$$

$$= \frac{2\sqrt{2}}{3}\ (1+\sqrt{3})^{\frac{1}{2}+\frac{1}{2}}\ (\sqrt{2-\sqrt{3}})$$ $\left[\text{Since } a^1 = a^{\frac{1}{2}}.\,a^{\frac{1}{2}}\right]$

$$= \frac{2\sqrt{2}}{3}\ \sqrt{1+\sqrt{3}}\ \ \sqrt{1+\sqrt{3}}\ \ \sqrt{2-\sqrt{3}}$$ $\left[\text{Since } a^{\frac{1}{2}} = \sqrt{a}\right]$

$$= \frac{2\sqrt{2}}{3}\ \sqrt{1+\sqrt{3}}\ \ \sqrt{(1+\sqrt{3})\,(2-\sqrt{3})}$$

$$= \frac{2\sqrt{2}}{3}\ \sqrt{1+\sqrt{3}}\ \ \sqrt{2-\sqrt{3}+2\sqrt{3}-3}$$

$$= \frac{2\sqrt{2}}{3} \sqrt{1+\sqrt{3}} \quad \sqrt{-1+\sqrt{3}}$$

$$= \frac{2\sqrt{2}}{3} \sqrt{\sqrt{3}+1} \quad \sqrt{\sqrt{3}-1}$$

$$= \frac{2\sqrt{2}}{3} \sqrt{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$= \frac{2\sqrt{2}}{3} \sqrt{(\sqrt{3})^2 - 1^2}$$

$(a + b)(a - b) = a^2 - b^2$

$$= \frac{2\sqrt{2}}{3} \sqrt{3-1}$$

$$= \frac{2\sqrt{2}}{3} \sqrt{2}$$

$$= \frac{2 \times 2}{3} = \frac{4}{3}$$

15. a

Let's find out the product of the given numbers

Consider (a)

$$(2 - \sqrt{3})(2 + \sqrt{3}) = 2^2 - (\sqrt{3})$$

$$= 4 - 3 = 1, \text{ a rational number}$$

i.e., option (a) is the answer

16. b

Consider the number 0, which is a whole number, but not a natural number. i.e., All whole numbers are not natural numbers.

17. c

..... - 3, -2, -1, are integers less than 0

i.e., the smallest integer cannot be determined.

18. a

The point P = the length OB

But ΔOAB is a right triangle. By Pythagoras theorem,

$$OB^2 = OA^2 + AB^2$$
$$= 2^2 + 2^2$$
$$= 4 + 4 = 8$$
$$\therefore OB = \sqrt{8} = 2\sqrt{2}$$

i.e., P is the point $2\sqrt{2}$

19. a

20. d

$$\frac{-4}{x} = \frac{x}{4}$$
$$-4(4) = x(x)$$
$$-16 = x^2$$
$$\therefore x = \sqrt{-16}$$

Square root of a negative number is not a real number. So the answer is option (d)

21. c

$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}$$

Consider the 1st term $\frac{1}{1+\sqrt{2}} = \frac{1}{1+\sqrt{2}} \quad \frac{1-\sqrt{2}}{1-\sqrt{2}}$

$$= \frac{1-\sqrt{2}}{1^2-(\sqrt{2})^2}$$

$$= \frac{1-\sqrt{2}}{1-2} = \frac{1-\sqrt{2}}{-1} = -(1-\sqrt{2})$$

$$\frac{1}{1+\sqrt{2}} = \sqrt{2}-1$$

$$\frac{1}{\sqrt{2}+\sqrt{3}} = \frac{1}{\sqrt{2}+\sqrt{3}} \quad \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

$$= \frac{\sqrt{2}-\sqrt{3}}{(\sqrt{2})^2-(\sqrt{3})^2}$$

$$= \frac{\sqrt{2}-\sqrt{3}}{2-3}$$

$$= \frac{\sqrt{2}-\sqrt{3}}{-1} = -(\sqrt{2}-\sqrt{3})$$

$$\frac{1}{\sqrt{2}+\sqrt{3}} = \sqrt{3}-\sqrt{2}$$

Similarly

$$\frac{1}{\sqrt{3}+\sqrt{4}} = \sqrt{4}-\sqrt{3}$$

$$\frac{1}{\sqrt{4}+\sqrt{5}} = \sqrt{5}-\sqrt{4}$$

$$\frac{1}{\sqrt{5}+\sqrt{6}} = \sqrt{6}-\sqrt{5}$$

$$\frac{1}{\sqrt{6}+\sqrt{7}} = \sqrt{7}-\sqrt{6}$$

$$\frac{1}{\sqrt{7}+\sqrt{8}} = \sqrt{8}-\sqrt{7}$$

$$\frac{1}{\sqrt{8}+\sqrt{9}} = \sqrt{9}-\sqrt{8}$$

$$\therefore \frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}} =$$

$$= \cancel{\sqrt{2}} - \mathbf{1} + \cancel{\sqrt{3}} - \cancel{\sqrt{2}} + \cancel{\sqrt{4}} - \cancel{\sqrt{3}} + \cancel{\sqrt{5}} - \cancel{\sqrt{4}} + \cancel{\sqrt{6}} - \cancel{\sqrt{5}} + \cancel{\sqrt{7}} - \cancel{\sqrt{6}} + \cancel{\sqrt{8}} - \cancel{\sqrt{7}} + \sqrt{\mathbf{9}} - \cancel{\sqrt{8}}$$

$$= -1+\sqrt{9}$$

$$= -1+3 = 2$$

22. a

The numbers are $\frac{3}{6}$, $\frac{9}{6}$, $\frac{1}{6}$ and $\frac{7}{6}$

Since the denominators are same, we can arrange these numbers in ascending order of numerators.

$\frac{1}{6}$, $\frac{3}{6}$, $\frac{7}{6}$, $\frac{9}{6}$

Least number $= \frac{1}{6}$

Greatest number $= \frac{9}{6}$

$\therefore$ percentage of least number in the greatest number percentage of $\frac{1}{6}$ in $\frac{9}{6}$ $= \left(\frac{\left(\frac{1}{6}\right)}{\left(\frac{9}{6}\right)} \times 100\right)\%$

$= \left(\frac{1 \times 6}{9 \times 6} \times 100\right)\%$

$= \left(\frac{1}{9} \times 100\right)\%$

$= \left(\frac{100}{9}\right)\% = 11\frac{1}{9}\%$

23. b

$$x = 3 + \sqrt{8}$$

$$= 3 + 2\sqrt{2} \qquad \text{Since } \sqrt{8} = 2\sqrt{2}$$

$$\frac{1}{x} = \frac{1}{3 + 2\sqrt{2}}$$

$$= \frac{1}{3 + 2\sqrt{2}} \times \frac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}}$$

$$= \frac{3 - 2\sqrt{2}}{3^2 - (2\sqrt{2})^2}$$

$$= \frac{3 - 2\sqrt{2}}{9 - 4(2)}$$

$$= \frac{3 - 2\sqrt{2}}{9 - 8}$$

$$\frac{1}{x} = 3 - 2\sqrt{2}$$

$$x^3 + \frac{1}{x^3} = (3 + 2\sqrt{2})^3 + (3 - 2\sqrt{2})^3$$

$$= 3^3 + \cancel{(2\sqrt{2})^3} + 3\,(3)^2\,\cancel{(2\sqrt{2})} + 3(3)\,(2\sqrt{2})^2 +$$

$$(3)^3 - \cancel{(2\sqrt{2})^3} - 3(3)^2\,\cancel{(2\sqrt{2})} + 3(3)\,(-2\sqrt{2})^2$$

$$= 27 + 9(4)(2) + 27 + 9(4)(2)$$

$$= 27 + 72 + 27 + 72$$

$$= 198$$

24 d

The given numbers are $\frac{4}{7}$, $\frac{1}{3}$, $\frac{2}{5}$ and $\frac{5}{9}$

We have to arrange these numbers in ascending order. So first find out the decimal expansions approximated to 2 digit of these numbers.

$$\frac{4}{7} = 0.57$$

$$\frac{1}{3} = 0.33$$

$$\frac{2}{5} = 0.4$$

$$\frac{5}{9} = 0.56$$

$0.33<0.4<0.56<0.57$

i.e., $\frac{1}{3} < \frac{2}{5} < \frac{5}{9} < \frac{4}{7}$

In ascending order, the numbers are

$\frac{1}{3}, \frac{2}{5}, \frac{5}{9}, \frac{4}{7}$

$\therefore$ Middle two rational numbers are $\frac{2}{5}$ and $\frac{5}{9}$

Average of middle two rational numbers

$$= \frac{\frac{2}{5} + \frac{5}{9}}{2}$$

$$= \frac{2(9) + 5(5)}{45(2)}$$

$$= \frac{18 + 25}{90}$$

$$= \frac{43}{90}$$

25. c

We have to simplify each expressions given in the options to identify the rationals.

$\pi - (-\pi) = \pi + \pi = 2.\pi$, an irrational number

$\sqrt{5} + \sqrt{4}$ which cannot be simplified further. It is an irrational number.

$$(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2}) = (\sqrt{3})^2 - (\sqrt{2})^2$$

$$= 3 - 2 = 1, \text{ a rational number}$$

i.e., Option (c) is not irrational

26. a

$$\sqrt[3]{4}\,.\sqrt[3]{44} = 4^{\frac{1}{3}}\,.\;4^{\frac{1}{3}}\,.\;11^{\frac{1}{3}}$$

$$= 4^{\frac{2}{3}}\,.\;11^{\frac{1}{3}}$$

$$= (4^2)^{\frac{1}{3}}\,11^{\frac{1}{3}}$$

$$= 16^{\frac{1}{3}}\,.\;11^{\frac{1}{3}}$$

$$= (16\,.\,11)^{\frac{1}{3}}$$

$$= (176)^{\frac{1}{3}}$$

$$= \sqrt[3]{176}$$

27. d

$$x = \frac{\sqrt{5} - 2}{\sqrt{5} + 2}$$

$$= \frac{\sqrt{5} - 2}{\sqrt{5} + 2} \quad \frac{\sqrt{5} - 2}{\sqrt{5} - 2}$$

$$= \frac{(\sqrt{5} - 2)^2}{(\sqrt{5})^2 - 2^2}$$

$$= \frac{(\sqrt{5} - 2)^2}{5 - 4}$$

$$= (\sqrt{5} - 2)^2$$

$$= (\sqrt{5})^2 - 2(\sqrt{5})(2) + 2^2$$

$$= 5 - 4\sqrt{5} + 4$$

$$x = 9 - 4\sqrt{5}$$

$$y = \frac{\sqrt{5}+2}{\sqrt{5}-2} = \frac{\sqrt{5}+2}{\sqrt{5}-2} \quad \frac{\sqrt{5}+2}{\sqrt{5}+2}$$

$$= \frac{(\sqrt{5}+2)^2}{5-4}$$

$$= (\sqrt{5}+2)^2$$

$$= (\sqrt{5})^2 + 2(\sqrt{5})(2) + 2^2$$

$$= 5 + 4\sqrt{5} + 4$$

$$y = 9 + 4\sqrt{5}$$

$$x^2 = (9 - 4\sqrt{5})^2$$

$$= 9^2 - 2(9)(4\sqrt{5}) + (4\sqrt{5})^2$$

$$= 81 - 72\sqrt{5} + 16(5)$$

$$x^2 = 161 - 72\sqrt{5} \text{ ------- } (1)$$

$$y^2 = (9 + 4\sqrt{5})^2$$

$$= 9^2 + 2(9)(4\sqrt{5}) + (4\sqrt{5})^2$$

$$= 81 + 72\sqrt{5} + 16(5)$$

$$= 81 + 72\sqrt{5} + 80$$

$$y^2 = 161 + 72\sqrt{5} \text{ ------- } (2)$$

$$xy = (9 - 4\sqrt{5})(9 + 4\sqrt{5})$$

$$= 9^2 - (4\sqrt{5})^2$$

$$= 81 - 16(5)$$

$$= 81 - 80$$

$$xy = 1 \quad \text{--------} \ (3)$$

$(1) + (2) + (3) \Rightarrow$

$$x^2 + y^2 + xy = 161 - 7\cancel{2\sqrt{5}} + 161 + 7\cancel{2\sqrt{5}} + 1$$

$$= 323$$

28. b

Given $$\frac{5 + 3\sqrt{2}}{5 - 3\sqrt{2}} = \frac{5 + 3\sqrt{2}}{5 - 3\sqrt{2}} \ \frac{5 + 3\sqrt{2}}{5 + 3\sqrt{2}}$$

$$= \frac{(5 + 3\sqrt{2})^2}{5^2 - (3\sqrt{2})^2}$$

$$= \frac{(5 + 3\sqrt{2})^2}{25 - 9(2)}$$

$$= \frac{(5 + 3\sqrt{2})^2}{25 - 18}$$

$$= \frac{(5 + 3\sqrt{2})^2}{7}$$

$$= \frac{5^2 + 2(5)(3\sqrt{2}) + (3\sqrt{2})^2}{7}$$

$$= \frac{25 + 30\sqrt{2} + 9(2)}{7}$$

$$= \frac{25 + 30\sqrt{2} + 18}{7}$$

$$= \frac{43 + 30\sqrt{2}}{7}$$

$$= \frac{43}{7} + \frac{30}{7}\sqrt{2}$$

Given $\frac{5 + 3\sqrt{2}}{5 - 3\sqrt{2}} = a + b\sqrt{2}$

i.e., $a + b\sqrt{2} = \frac{43}{7} + \frac{30}{7}\sqrt{2}$

Equating the coefficient of $\sqrt{2}$,

$$\frac{30}{7} = b$$

Equating the constant terms,

$$\frac{43}{7} = a$$

29. b

Consider $\sqrt{18} - \sqrt{50}$

The conjugate pair $\sqrt{18} + \sqrt{50}$ is a rationalising factor. To find the least rationalising factor, we have to simply the given expressions,

$\sqrt{18} - \sqrt{50}$

$\sqrt{18} = \sqrt{2 \text{ x } 3 \text{ x } 3} = 3\sqrt{2}$

$\sqrt{50} = \sqrt{2 \text{ x } 5 \text{ x } 5} = 5\sqrt{2}$

$\therefore \sqrt{18} - \sqrt{50} = 3\sqrt{2} - 5\sqrt{2} = -2\sqrt{2}$

In the simplified expression, the rational number $\sqrt{2}$ is present. To rationalise this, multiply it by $\sqrt{2}$. i.e., the rationalising factor is $\sqrt{2}$ which is also least.

30. d

$$\frac{22}{2\sqrt{3}+1} + \frac{17}{2\sqrt{3}-1}$$

Consider the first term,

$$\frac{22}{2\sqrt{3}+1} = \frac{22}{2\sqrt{3}+1} \quad \frac{2\sqrt{3}-1}{2\sqrt{3}-1} = \frac{22\,(2\sqrt{3}-1)}{(2\sqrt{3})^2 - 1^2}$$

$$= \frac{22\,(2\sqrt{3}-1)}{4(3)-1}$$

$$= \frac{22\,(2\sqrt{3}-1)}{12-1}$$

$$= \frac{22\,(2\sqrt{3}-1)}{11}$$

$$= 2\,(2\sqrt{3}-1)$$

$$\frac{22}{2\sqrt{3}+1} = 4\sqrt{3} - 2$$

Consider the 2nd term,

$$\frac{17}{2\sqrt{3}-1} = \frac{17}{2\sqrt{3}-1} \quad \frac{2\sqrt{3}+1}{2\sqrt{3}+1}$$

$$= \frac{17(2\sqrt{3}+1)}{(2\sqrt{3})^2 - 1^2}$$

$$= \frac{17(2\sqrt{3}+1)}{4(3) - 1}$$

$$= \frac{17(2\sqrt{3}+1)}{12 - 1}$$

$$= \frac{17(2\sqrt{3}+1)}{11}$$

$$= \frac{34\sqrt{3}+17}{11}$$

$$= \frac{34}{11}\sqrt{3} + \frac{17}{11}$$

$$\frac{22}{2\sqrt{3}+1} + \frac{17}{2\sqrt{3}-1} = 4\sqrt{3} - 2 + \frac{34}{11}\sqrt{3} + \frac{17}{11}$$

$$= \left(4 + \frac{34}{11}\right)\sqrt{3} + \frac{17}{11} - 2$$

$$= \frac{44+34}{11}\sqrt{3} + \frac{17-22}{11}$$

$$= \frac{78}{11}\sqrt{3} + \frac{-5}{11}$$

$$= \frac{78\sqrt{3}-5}{11}$$

PRACTICE TEST

"Practice makes perfect". Test the level of your accuracy and speed by answering the given problems. Please note the time you spend for each problem. Write all the necessary steps. But if you are thorough with the ideas you can skip some steps. Please send the answers to our office in the business envelope which is free of cost. We will provide you proper guidance. If you want to get a proper evaluation and assessment, you have to do it sincerely.

1. Which of the following is not a rational number?

   a) $\sqrt{9}$ b) $\sqrt{16}$ c) $-\sqrt{4}$ d) $2\sqrt{2}$

2. What is the decimal equivalent of $\left(\frac{1}{3}+\frac{1}{9}\right)$?

   a) $0.\overline{4}$ b) $0.\overline{3}$ c) 0.21 d) $1.\overline{4}$

3. Which of the following statement is true?

   a) $\frac{5}{7} < \frac{7}{9} < \frac{9}{11} < \frac{11}{13}$

   b) $\frac{11}{13} < \frac{9}{11} < \frac{7}{9} < \frac{5}{7}$

   c) $\frac{5}{7} < \frac{11}{13} < \frac{7}{9} < \frac{9}{11}$

   d) $\frac{5}{7} < \frac{9}{11} < \frac{11}{13} < \frac{7}{9}$

4. If $x = 7 + 4\sqrt{3}$ and $xy = 1$, then $\frac{1}{x^2} + \frac{1}{y^2} =$

   a) 64 b) 134 c) 194 d) 234

5. The rationalising factor of $2\sqrt{3} - \sqrt{7}$ is

   a) $\sqrt{3} + \sqrt{7}$ b) $2\sqrt{3} + \sqrt{7}$ c) $\sqrt{3} + 2\sqrt{7}$ d) none of these

6. The greatest amoung $3\sqrt{9}$, $4\sqrt{11}$, $6\sqrt{17}$ is

a) $3\sqrt{9}$ b) $4\sqrt{11}$ c) $6\sqrt{17}$ d) Cannot be determined

7. If $x = \frac{1}{3-\sqrt{8}}$ and $y = \frac{1}{3+\sqrt{8}}$, then xy =

a) 17 b) $\frac{1}{3+\sqrt{8}}$ c) 1 d) none of these

8. If $\frac{3}{\sqrt{3}+\sqrt{2}} = a\sqrt{3} - b\sqrt{2}$ then

a) a = 3, b = -3 b) a = -3, b = 3

c) a = 3, b = 2 d) a = 3, b = -2

9. The least rationalising factor of $2\sqrt{125}$ is

a) $\sqrt{125}$ b) $\sqrt{2}$ c) $\sqrt{5}$ d) $\sqrt{25}$

10. If $x = 7+\sqrt{3}$, $xy = 4$ then $x^4 + y^4$

a) 400 b) 368 c) 352 d) 200

11. If $a = \sqrt{21} - \sqrt{20}$ and $b = \sqrt{18} - \sqrt{17}$ then

a) a = b b) a + b = 0 c) a > b d) a < b

12. Which of the following numbers does not lie between -1 and 1?

a) $\frac{3}{4}$ b) $\frac{-2}{3}$ c) 2 - 1 d) $\frac{-5}{4}$

13. Rationalising factor of $a^{\frac{1}{3}} + a^{\frac{-1}{3}}$

a) $a^{\frac{1}{3}} - a^{\frac{-1}{3}}$ b) $a^{\frac{2}{3}} + a^{\frac{-2}{3}}$ c) $a^{\frac{2}{3}} - a^{\frac{2}{3}}$ d) $a^{\frac{2}{3}} + a^{\frac{-2}{3}} - 1$

14. The product of which of the following pairs of numbers will be a rational number?

a) $2 - \sqrt{3}, 2 + \sqrt{3}$ b) $3 - \sqrt{3}, 4 - \sqrt{3}$

c) $2 - \sqrt{2}, 2 + \sqrt{7}$ d) $1 - \sqrt{3}, \sqrt{7}$

15. Which of the following statement is not true?

a) Every real number is represented by a unique point on the number line

b) Every point on the number line represents a unique real number.

c) Irrational numbers cannot be represented on the number line.

d) There are infinity many numbers on the number line.

16. What is the fractional equivqlent of 0.36?

a) $\frac{11}{25}$ b) $\frac{25}{36}$ c) $\frac{9}{25}$ d) $\frac{36}{25}$

17. To what set of numbers does $\sqrt{13}$ belong?

a) irrationals b) integers c) whole numbers d) rationals

18. Which is the smallest integer?

a) 1 b) 0 c) -1 d) cannot be determined

19. Which of the following is an irrational number?

a) $\frac{\sqrt{5} - 2}{\sqrt{5} + 2} \times \frac{\sqrt{5} + 2}{\sqrt{5} - 2}$ b) $(3 - \sqrt{8})(3 + \sqrt{8})$

c) $5 - 4\sqrt{6} + \frac{1}{5 - 4\sqrt{6}}$ d) $3\ 6\sqrt{50} \div 48\sqrt{8}$

20. In the expression $\frac{p}{q}$, p and q are integers q ≠ 0, and q = $2^n \times 5^m$, where n and m are natural numbers, then

a) the decimal expansion of $\frac{p}{q}$ terminates

b) $\frac{p}{q}$ is a non recurring decimal.

c) $\frac{p}{q}$ is a non terminating non recurring decimal

d) Can't say

Project

Write a note about golden number. Is it an irrational number?

Most people are familiar with the number, π, since it is one of the most common irrational numbers known to man. But there is another irrational number that has the same propensity for popping up which is not as well known as π. This wonderful number is **golden number**, a Greek letter ϕ (read as phi) is used to denote this number.

Consider the solutions of the equation $x^2 - x - 1 = 0$

This is a quadratic equation

$$\therefore \text{Solutions } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-(-1) \pm \sqrt{1 + 4}}{2}$$

$$= \frac{+1 \pm \sqrt{5}}{2}$$

a = coefficient of x^2
= 1
b = coefficient of x
= -1
c = constant
= -1

i.e., roots are, $x = \frac{1+\sqrt{5}}{2}$ or $x = \frac{1-\sqrt{5}}{2}$

We consider the first root to be the golden number (ϕ)

$\therefore$ **Golden number ϕ** $= \frac{\mathbf{1+\sqrt{5}}}{\mathbf{2}}$

We know $\sqrt{5}$ is an irrational number,

$\therefore \frac{1+\sqrt{5}}{2}$ is an irrational number.

$\Rightarrow$ Golden number is an irrational number.

Let's discuss these in detail

Consider $\phi = \frac{1+\sqrt{5}}{2}$

We know $\sqrt{5} \sim 2.236$

$$\therefore \phi = 1 + \frac{2.236}{2} = 1.618$$

$$\phi \cong 1.618$$

If 2 objects are in the ratio 1.618, we say that they are in golden proportion, or 1.618 is known as golden ratio.

Golden ratio in geometrical figures

Draw a rectangle with length 5.35 and breadth 3.33 approximately

$\frac{l}{b} \sim 1.618$

If the ratio of length and breadth is ϕ, then that rectangle is known as golden rectangle.

Golden ratio can be seen in other geometrical figures also.

Golden ratio in Nature

Golden ratio is most pleasing to eyes. In nature, so many flowers exhibit golden ratio.

For example, Consider the head of a daisy, one can discover that the individual florets of the daisy (and of a sunflower as well) grow in two spirals extending out from the centre. The first spiral has 21 arms, while the other has 34. $\therefore$ thier ratio $\frac{34}{21} \sim 1.618$. i.e., they exhibit golden ratio.

Golden ratio can be found in patterns on butterflies' wings.

We can observe ϕ in many things.

The golden ratio, also known as the divine proportion. Though it is an irrational number, it is most pleasing to human eyes.

Puzzles

Do you know this magic?

If you perform the following operations, you will always arrive at 100.

Always 100

- **Let's think a single digit number other than zero**
- **Add 99 to it**
- **Strike out the last digit**
- **Add 5 to the result**
- **Multiply it by 7**
- **Subtract 5 from it**

- A cross word puzzle for you

■	3			2	8	■	■	■
■	■	■	■			■	6	■
1				4		■		■
	■	■	5					
	■	10	9					■
	■	■			■	■		■
7					■	■		■
	■	■	12					
11								■

Across

1. An angle greater than 180^0 but less than 360^0 is called ____angle.
3. Additive identity is
5. 100 is _____ of 10
7. The smallest prime number is.
10. An angle less than 90^0 is called
11. ________ Numbers is used for counting.
12. A whole number.

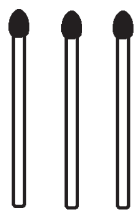
Down

1. ________ angle has its own supplement.
2. $(999)^0$ = _______
3. The letter used to denote integers

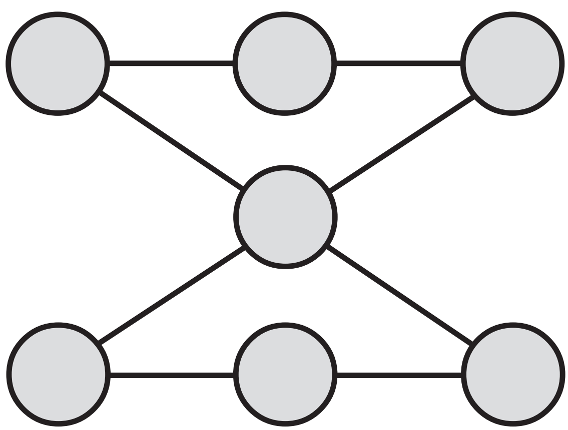
4. Vertically opposite angles are ________
6. The never intersecting lines are called ______
7. The first power of 10 is
8. How many zeroes are there in 10^6?
9. 3^{rd} power of a number is also known as

- **Matchstick Puzzle**

Move two matchsticks to change three into six



- **A prime Number Game :**



Here are seven prime numbers 5, 7, 11, 13, 17, 19, 23. Can you arrange these prime numbers in the seven circles so that the rows and diagonals add up to the same prime number?